
**Measurement of fluid flow rate in
closed conduits — Radioactive tracer
methods**

*Mesure du débit des fluides dans des conduites fermées — Méthodes
par traceur radioactif*

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Foreword

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The procedures used to develop this document and those intended for its further maintenance are described in the ISO/IEC Directives, Part 1. In particular, the different approval criteria needed for the different types of ISO documents should be noted. This document was drafted in accordance with the editorial rules of the ISO/IEC Directives, Part 2 (see www.iso.org/directives).

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Any feedback or questions on this document should be directed to the user's national standards body. A complete listing of these bodies can be found at www.iso.org/members.html.

Introduction

The accurate knowledge of fluid flow rates (liquid and gas) in industrial systems is an essential requirement of processing industries. The fluid flow rate measurement is usually needed for various reasons i.e. calibration of installed flow meters, fluid balance, measurement of efficacy of pumps or turbines, distribution of flow in a network of pipes, etc. Generally, the industrial systems where the flow rates are needed to be measured are classified into two categories, namely open channels and closed conduits. Closed conduits are conveyance systems where the flow of fluid is confined on all boundaries (i.e. pipe systems) while open channels are systems where the stream has a free surface open to the atmosphere such as canals, rivers, streams, sewer lines, effluent channels, partly filled conduits.

This document deals with single phase fluid flow rate measurement in closed conduits only, by using the radioactive tracer method. Flow in closed conduits is caused by an axial pressure difference. Various types of flow meters, such as ultrasonic, electromagnetic, acoustic, Venturi, Pitot tube, and gamma transmission, are routinely used for flow rate measurements in closed conduits in industry. The selection of a suitable method for a particular application depends on the type and nature of the system, physical properties of the flowing fluid, flow patterns of the fluid, limitations imposed by the design and operating condition of the plant, cost and installation of the equipment. One advantage of the radioactive tracer methods is that measurement can be carried out online in harsh industrial environment, from outside of the conduits while the process is in operation, with no disruption, and with a high accuracy. This document treats radioactive tracer methods only.

The use of radioactive tracer methods for the measurement of fluid flow rates in closed conduits is one of the most common and well-established application of the radioactive tracer technology in industry. The major methods that have been found to be particularly applicable for online flow rate measurement and flowmeter calibration are the pulse velocity or transit time method, as well as dilution methods, known as constant rate injection method and integration method.

This document is developed to fill the need for a generalized reference based on fundamental principles to measure fluid flow using radioactive tracer methods.

For single phase steady-state flow of fluid in a closed conduit, the volumetric flow rate can be measured using this method. If the mass density is known, the mass flow rate can be deduced from the volume flow rate.

The accuracy of flow rate measurement with the radioactive tracer methods depends on how well the injected tracer material mixes with the flowing fluid before the measuring section. It depends on the amount of tracer injected and the accuracy of the measurement devices.

Measurement of fluid flow rate in closed conduits — Radioactive tracer methods

1 Scope

This document defines the measurement of single phase fluid flow rate in closed conduits using radioactive tracer methods.

2 Normative references

There are no normative references in this document.

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

ISO and IEC maintain terminology databases for use in standardization at the following addresses:

- ISO Online browsing platform: available at <https://www.iso.org/obp>
- IEC Electropedia: available at <https://www.electropedia.org/>

3.1

closed conduit

conveyance system where the flow of fluid is confined on all boundaries (i.e. pipe systems)

3.2

mixing length

l_m

shortest distance at which the variation in concentration of the tracer over the cross-section is less than some pre-determined value (for example 0,5 %)

4 Principles of radioactive tracer methods

4.1 General

Three radioactive tracer methods for flow rate measurement have been used:

- transit time method;
- constant rate injection method;
- integration method.

Both the constant rate injection method and the integration method are part of the dilution methods. Radiation dose considerations are given in [Annex B](#).

4.2 Transit time method

4.2.1 Principle

In the transit time method, a quantity of a radioactive tracer is injected instantaneously into a flowing stream. Two detection cross-sections downstream the injection cross-section are commonly used

for registration of the gamma radiation emitted from the radioactive tracer in the flow. Both cross-sections are sufficiently far from the injection cross-section to allow adequate (homogeneous) mixing of the tracer with the fluid flow. Each detector registers a response curve when the tracer cloud crosses the detection cross-section. The two response curves are compared to provide the transit time of the tracer or fluid between the two detection cross-sections. Under these conditions, the volumetric flow rate Q is given by [Formula \(1\)](#):

$$Q = \frac{V}{\bar{t}} \quad (1)$$

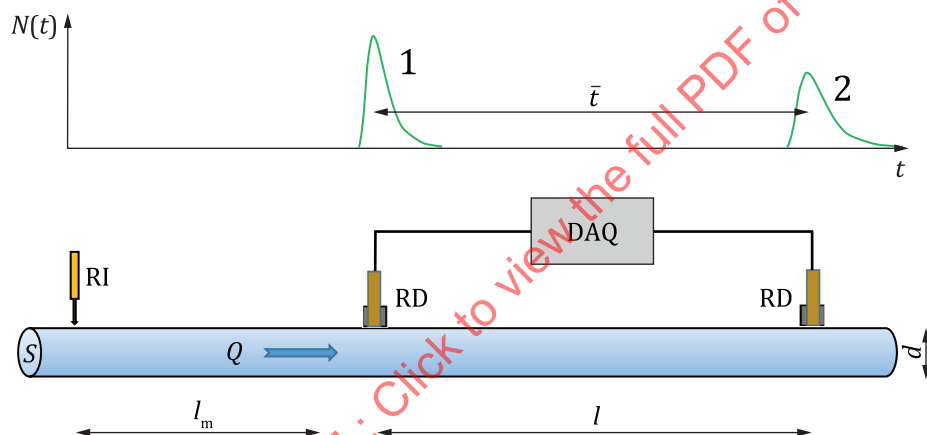
where

Q is the volumetric flow rate;

V is the volume of the conduit between the detection cross-sections;

\bar{t} is the transit time of the tracer between the two detection cross-sections.

The value of \bar{t} is obtained by measuring the difference in the centre of gravity, i.e. the first moment of the two response curves as shown in [Figure 1](#).



Key

- 1 response curve of 1st detector
- 2 response curve of 2nd detector
- t time
- \bar{t} transit time of the tracer between the two detection cross-sections
- Q volumetric flow rate
- DAQ data acquisition system
- S area of across-section
- RI radioactive tracer injector
- RD radiation detector
- l_m mixing length
- l distance between the two detection cross-sections
- d inner diameter of conduit
- $N(t)$ measured radiation count rate

Figure 1 — Principle of transit time method

The volume of the measuring section, V , and the volume flow rate, Q , are given respectively by [Formulae \(2\)](#) and [\(3\)](#):

$$V = S \cdot l \quad (2)$$

$$Q = \frac{V}{\bar{t}} = \frac{S \cdot l}{\bar{t}} \quad (3)$$

where

V is the volume of the conduit between the detection cross-sections;

S is the cross-section of the fluid in the closed conduit;

l is the distance between the two detection cross-sections;

\bar{t} is the transit time of the tracer between the two detection cross-sections.

The transit time, \bar{t} , is calculated by the difference in the mean residence times (first moments) between the two response curves, as given by [Formula \(4\)](#):

$$\bar{t} = \tau_2 - \tau_1 = \frac{\sum t_{2i} n_{2i}}{\sum n_{2i}} - \frac{\sum t_{1i} n_{1i}}{\sum n_{1i}} \quad (4)$$

where

\bar{t} is the transit time of the tracer between the two detection cross-sections;

τ is the mean residence time, and the indexes 1 and 2 refer to the 1st and 2nd detector, respectively;

n is the corrected count per count time, and the indexes 1 and 2 refer to the 1st and 2nd detector, respectively.

4.2.2 Special recommendation for the transit time method

For this method, a conduit length of constant cross-section between the two detection cross-sections should be ensured, so that the flow parameters are constant over the measuring length. The internal volume of the measuring section shall be determined with sufficient accuracy.

4.2.3 Advantages of transit time method

The radioactive tracer transit time method seems to provide the most effective field calibration method for flow rate measurement in closed conduits. It is suitable for both liquid and gas flows and covers a large range of flow rates with a small uncertainty (see [Annex A](#)).

The main advantages of this method are as follows:

- it is only necessary to determine the response curve at two detection cross-sections;
- it is not necessary to know activity, volume or flow rates of the injected radioactive tracer;
- it is not necessary to collect any samples;
- the activity of the radioactive tracer used by this method is considerably smaller than needed for other methods.

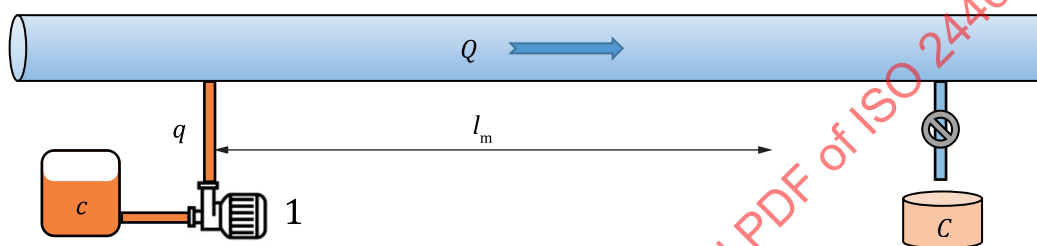
4.3 Constant rate injection method

4.3.1 Principle

The constant rate injection method is based on the principle of conservation of tracer activity. A tracer of activity concentration c is injected with constant volume flow rate q to the main flow with volume flow rate Q . As there is no gain or loss of tracer in the measuring section, the injected tracer shall eventually appear with the same total activity (but with a different activity concentration, C , because of dilution in the main flow) at any downstream detection point, as given by [Formula \(5\)](#):

$$q \times c = Q \times C \quad (5)$$

[Figure 2](#) shows the principle of constant rate injection method.



Key

- 1 constant rate injection pump
- Q volumetric flow rate in closed conduit
- q volumetric flow rate of tracer solution
- l_m mixing length
- c activity concentration of tracer solution
- C activity concentration of collected sample

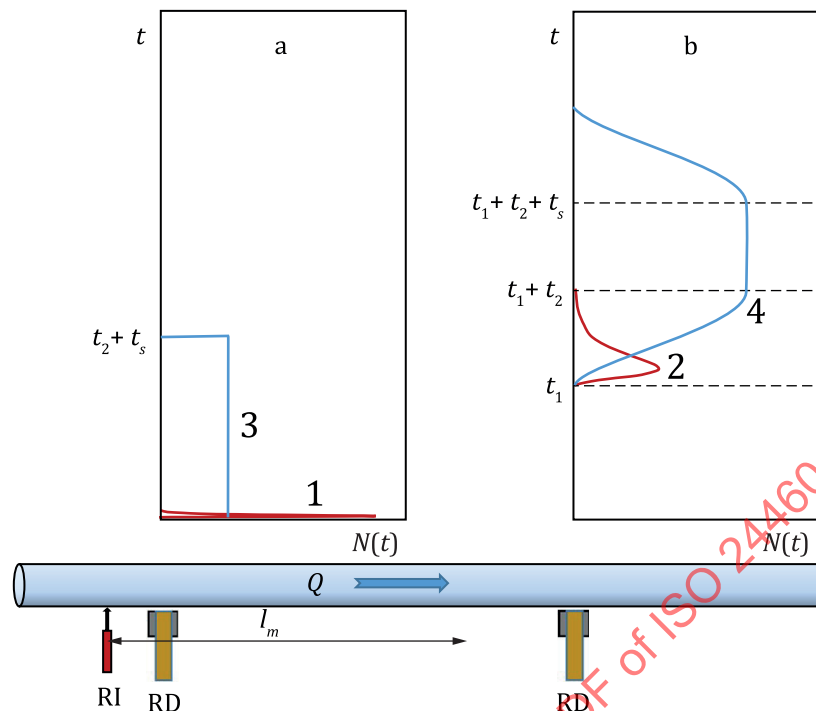
Figure 2 — Principle of constant rate injection method

The volumetric flow rate of the mainstream can be calculated by [Formula \(6\)](#):

$$Q = q \frac{c}{C} \quad (6)$$

4.3.2 Duration of injection

The duration of injection shall be such that stable concentration conditions are established at all points of the sampling cross-section over a sufficient period of time. A suitable duration of injection may be determined by a preliminary investigation involving a pulse injection of a radioactive tracer. The response curve (curve 2 of [Figure 3](#)) of the pulse injection is obtained at the sampling cross-section. The curve starts at time t_1 after the injection and the duration of the curve is t_2 .

**Key**

- 1 pulse injection curve
- 2 response curve of the pulse injection
- 3 imaginary constant rate injection curve
- 4 response curve of the imaginary constant rate injection
- Q volumetric flow rate
- t time
- t_s time for sampling
- $N(t)$ measured radiation count rate
- RI radioactive tracer injector
- RD radiation detector
- l_m mixing length
- a Tracer injection curves at injection point.
- b Tracer response curves at measuring point.

Figure 3 — Determination of the duration of injection

In constant rate injection method, if it is required to achieve steady conditions for a period of time t_s in a selected sampling cross-section, it is necessary to keep the constant rate injection for a period of time $t_2 + t_s$. Then, the measurement (detection or sampling) can be performed from the time $t_1 + t_2$ to time $t_1 + t_2 + t_s$, after the start of the constant rate injection.

4.3.3 Advantage of the constant rate injection method

The main advantage of this method is:

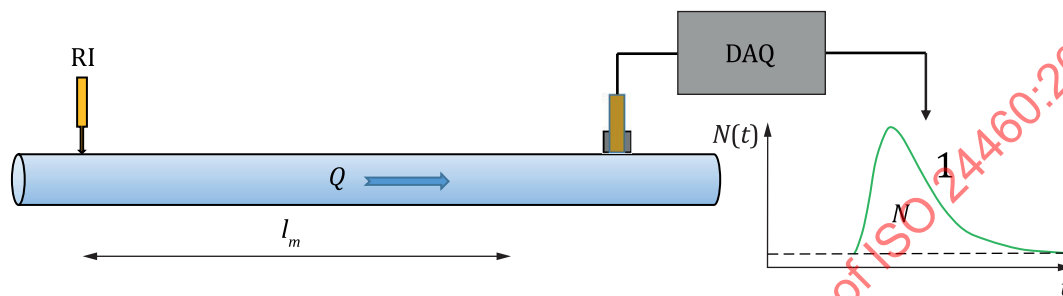
- that it is not necessary to know the geometrical characteristics of the conduit.

4.4 Integration method

4.4.1 Principle

In integration (or total-count) method, it is assumed that if an activity A of tracer is injected, then this amount — as there is no gain or loss of tracer in the measuring section — shall eventually pass any downstream detection cross-section.

Pulse injection of tracer into closed conduit is applied. The flow rate is determined from the cumulative response of a detector located externally on the closed conduit. [Figure 4](#) shows the principle of the integration method.



Key

1	response curve of injected radioactive tracer
RI	radioactive tracer injector
Q	volumetric flowrate
l_m	mixing length
DAQ	data acquisition system
$N(t)$	measured radiation count rate
N	integrated net radiation count
t	time

Figure 4 — Principle of integration method

The integrated net radiation count, N , (corrected for background and decay) is registered, then the flow rate Q is given by [Formula \(7\)](#)

$$Q = F \frac{A}{N} \quad (7)$$

where

A is the total injected activity [Bq];

F is the calibration factor relating A to N [counts per unit time per Bq/l];

N is the integrated net radiation count.

The calibration factor, F , [counts per unit time per unit activity concentration] shall be determined beforehand. For the calibration of detector placed external to a closed conduit a section of identical conduit shall be set up. This section shall be longer than the field of view of the collimated detector. Then, the net radiation count rate of an identically located detector to a known activity concentration within the conduit shall be measured.

4.4.2 Advantages of the integration method

The main advantages of the integration method compared to the constant rate injection method are:

- a smaller amount of radioactive tracer can be used;
- less field operation time is needed.

5 Choice of radioactive tracer

5.1 General

5.1.1 Requirements

The radioactive tracer shall comply with the following requirements:

- have identical flow behaviour as the fluid being traced;
- mix easily and homogeneously with the fluid in the conduit;
- be measurable with sufficient sensitivity;
- have a suitable half-life for the examination;
- be sufficiently chemically stable under the conditions of use;
- be affordable.

5.1.2 Radioactive tracers

Tables 1 and 2 present the commonly used radioactive tracers for measurement of fluid flow rate. Only gamma ray emitting tracers are considered here.

Table 1 — Tracers labelled with radionuclides produced in nuclear reactors or particle accelerators

Radionuclide	Half-life	Gamma energy in keV (Probability in %)	Chemical form of tracer/ form of carrier compound	Traced phase
Sodium-24 (^{24}Na)	15 h	1 368,6 (100) 2 754,0 (100)	$^{24}\text{Na}^+$ /Sodium carbonate, Na_2CO_3 or Sodium bicarbonate, NaHCO_3	Aqueous
Bromine-82 (^{82}Br)	36 h	554,5 (71,7)	$^{82}\text{Br}^-$ /Ammonium bromide, NH_4Br	Aqueous
		619,1 (43,7) 698,4 (28,4)	Radiolabelled p-dibromobenzene, $\text{C}_6\text{H}_4^{82}\text{Br}_2/\text{C}_6\text{H}_4\text{Br}_2$	Organic
		776,5 (83,6) 1 044,0 (25,6) 1 317,5 (26,9)	$\text{CH}_3^{82}\text{Br}$ /Methyl bromide, CH_3Br	Gases
Iodine-123 (^{123}I)	13,2 h	159,0 (83,3)	$^{123}\text{I}^-$ /Potassium iodide, KI or sodium iodide, NaI	Aqueous
			Radiolabelled iodobenzene, $\text{C}_6\text{H}_5^{123}\text{I}/\text{C}_6\text{H}_5\text{I}$	Organic
Iodine-131 (^{131}I)	8,03 d	364,5 (81,5)	$^{131}\text{I}^-$ /Potassium iodide, KI or sodium iodide, NaI	Aqueous
			Radiolabelled iodobenzene, $\text{C}_6\text{H}_5^{131}\text{I}/\text{C}_6\text{H}_5\text{I}$	Organic

Table 1 (continued)

Radionuclide	Half-life	Gamma energy in keV (Probability in %)	Chemical form of tracer/ form of carrier compound	Traced phase
Xenon-133 (¹³³ Xe)	5,27 d	45,0 (52,8) 81,0 (36,9)	¹³³ Xe ⁰ /Xenon	Gases
Krypton-79 (⁷⁹ Kr)	35 h	261,3 (12,7) 511,0 (14,0)	⁷⁹ Kr ⁰ /Krypton	Gases
Argon-41 (⁴¹ Ar)	109,6 min	1 293,6 (99,22)	⁴¹ Ar ⁰ /Argon	Gases

Table 2 — Radiotracers produced from radionuclides eluted from radionuclide generators

Radionuclide generator	Half-life mother/ half-life daughter	Gamma energy in keV (Probability in %)	Chemical form of tracer ^{a b c d e}	Traced phase
⁶⁸ Ge → ⁶⁸ Ga	271 d/67,7 min	511,0 (177,8), 1 077,3 (3,2)	⁶⁸ Ga ³⁺ , ⁶⁸ Ga-[DOTA] ⁻	Aqueous
^{99m} Mo → ^{99m} Tc	66 h/6 h	140,5 (89,0)	^{99m} TcO ₄ ⁻	Aqueous
¹¹³ Sn → ^{113m} In	115 d/99,5 min	391,7 (64,9)	^{113m} In ³⁺ , ^{113m} In-[EDTA] ⁻	Aqueous
			^{113m} In-[D2EHPA]	Organic
¹³⁷ Cs → ^{137m} Ba	30 y/2,55 min	661,7 (89,9)	^{137m} Ba ²⁺ , ^{137m} Ba-[EDTA] ⁿ⁻	Aqueous
			^{137m} Ba-[DC18C6][HDNNS] ₂	Organic

^a DOTA = 1,4,7,10-tetraazacyclododecane-1,4,7,10-tetraacetic acid.
^b EDTA = ethylene-diamine tetra-acetic acid.
^c D2EHPA = di(2-ethylhexyl) phosphoric acid.
^d DC18C6 = di-cyclohexano-18-crown-6 or 2,3,11,12-dicyclohexano-1,4,7,10,13,16-hexaoxacyclo-octadecane.
^e HDNNS = di-nonyl naphthalene sulfonic acid.

5.2 Advantages of radioactive tracers

The main advantages of employing radioactive tracer are:

- these can be detected by means of detectors located outside the conduit (for tracers emitting sufficiently energetic gamma radiation);
- with short half-life radioactive tracers, any contamination danger disappears quickly and there is no permanent pollution.

5.3 Particular advantages of radionuclide generators

The main advantage of using radionuclide generator is, that practically useable quantity of radioactive tracer of short half-life is available repeatedly at the measuring place.

5.4 Selection of radioactive tracer

5.4.1 Type of emitted radiations

Gamma ray emitting tracers are preferred to beta ray emitting tracers because the measurement of this type of radiation can be made through conduit walls and the self-absorption of radiation by the fluid is decreased.

5.4.2 Half-life

The transit time method makes it possible to use radioactive tracers with much shorter half-lives than those required for constant rate injection and integration methods. A radioactive tracer shall be chosen with the shortest possible half-life consistent with the above-mentioned conditions and with the conditions of supply, storage and measurement of the radionuclide, in order to minimize any effect of contamination and unnecessary radiological exposure associated with the handling of the tracer.

6 Choice of adequate mixing length

6.1 General

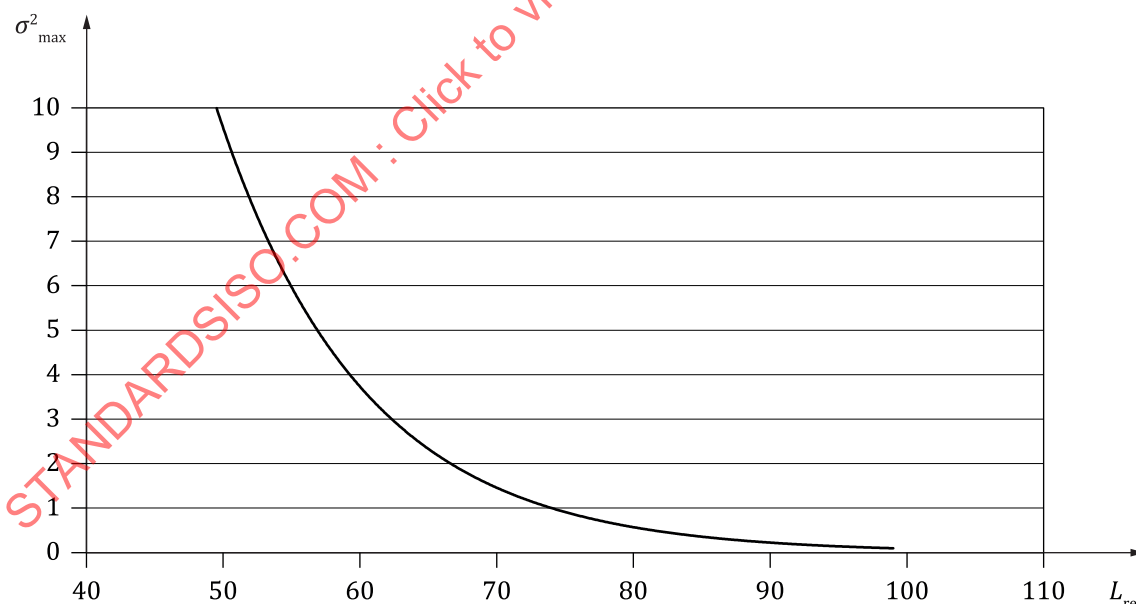
When a radioactive tracer is used to measure the flow of fluid in a conduit, there should be sufficient distance between the injection cross-section and the detection or sampling cross-section.

Mixing length is defined as the distance beyond which the variation in the conduit cross-section of tracer concentration is smaller than some previously chosen value. Therefore, the mixing length is not a fixed value but varies according to the allowed concentration variations in the conduit cross-section: the smaller the acceptable variation the greater the mixing length.

6.2 Consideration on the mixing length

6.2.1 General

The mixing length varies with Reynolds number Re , conduit friction and the injection technique employed. [Figure 5](#) shows variation in cross-section homogeneity of tracer as function of relative mixing length for a Reynold number $Re = 10^5$ for a smooth pipe.



Key

- L_{rel} relative mixing length (ratio between length and diameter of pipe)
- σ^2_{max} maximum variation of tracer concentration in conduit cross-section (%)

Figure 5 — Effect of variation in cross-section homogeneity of tracer on relative mixing length for a smooth pipe

The length of conduit between the injection and first detector shall be equal to or greater than the mixing length and should preferably contain no pipe fittings or sections likely to significantly increase the longitudinal dispersion of the tracer at the detection cross-sections.

6.2.2 Examples of injection techniques for reducing mixing length

6.2.2.1 General

Experience has shown that good cross-sectional mixing, for central injection, may require as many as 100 pipe diameters downstream the injection cross-section to be achieved. It is often not possible to inject the radioactive tracer at such a distance upstream of the measurement section. Therefore, it is required to reduce that length by using appropriate radioactive tracer injection techniques and devices, or to accept lower accuracy of the results of flow rate.

6.2.2.2 Multiple-orifice injectors

Substantial reduction in mixing length can be obtained by injecting the tracer through multiple orifices uniformly distributed on the conduit wall or (if possible) concentrically inside the conduit. [Figure 6](#) shows variation in cross-section homogeneity of tracer as function of relative mixing length for four different types of injection.

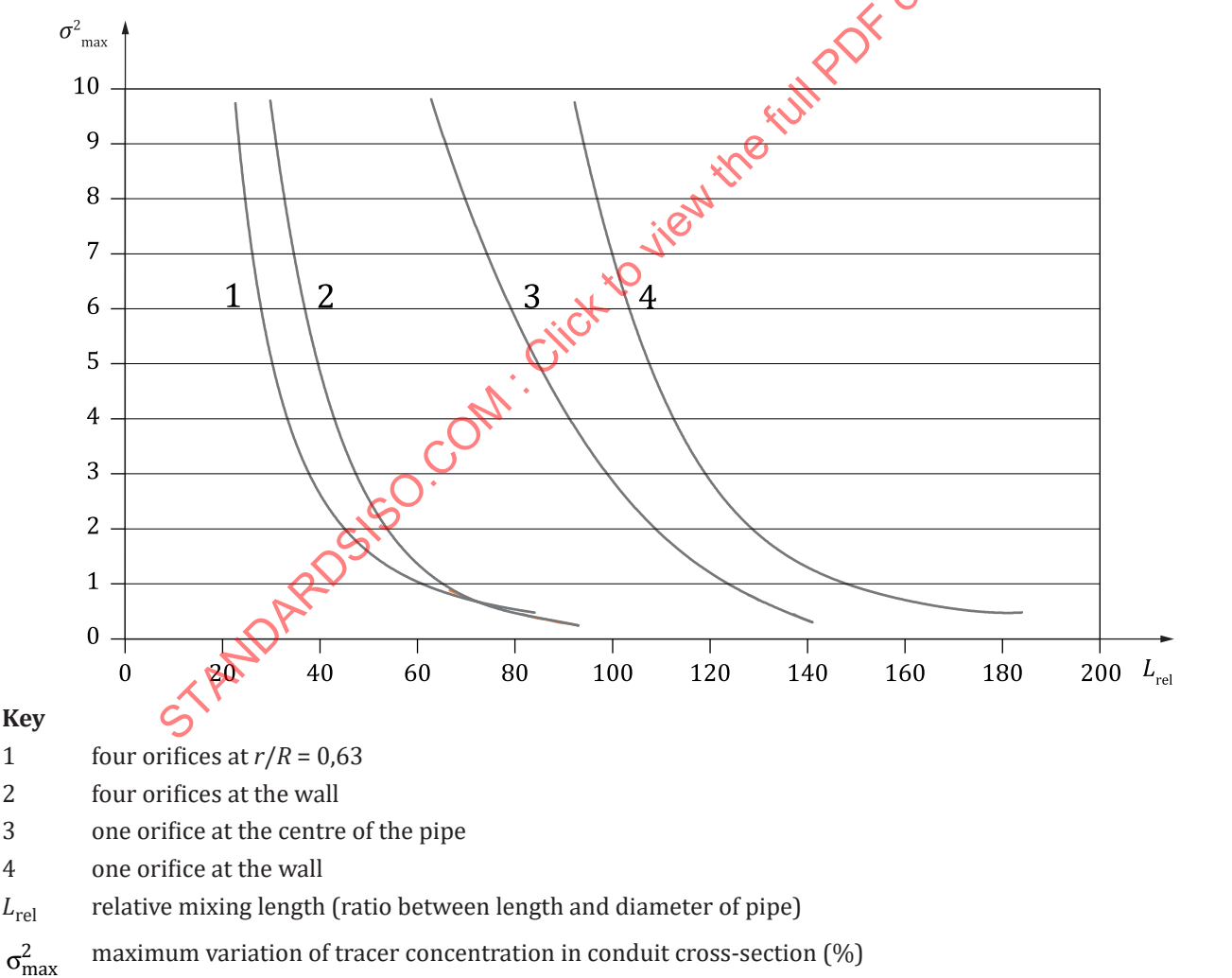


Figure 6 — Effect of variation in cross-section homogeneity of tracer on relative mixing length for four different types of injection

If multi-orifice injections are used, the device shall be designed so as to allow a simultaneous injection with equal injection rate in every point.

6.2.2.3 High velocity jets

Injecting the tracer counter-currently at a velocity much larger than bulk flow velocity induces high mixing at the end of the jet. The reduction in good mixing length depends on the number and momentum of the jets and on their angle with respect to the main flow direction. A simple jet arrangement can bring about 30 % reduction in the mixing length as compared to the single central injection point.

6.2.2.4 Vortex mixing

Incorporating obstacles within the conduit, just after tracer injection cross-section, produces turbulence that enhances mixing and reduces the mixing length. As an example, injecting the tracer through three triangular plates, at an angle of 40° to the main flow direction, reduces the mixing length by one third with respect to a central single injection point.

6.2.2.5 Pumps and turbines

If tracer is injected upstream of a pump or a turbine, the mixing length is considerably reduced. Centrifugal pumps reduce the mixing length by about 50 pipe diameters.

6.2.2.6 Bends, valves and other obstacles

Every singularity in the conduit promotes turbulence that tends to decrease good mixing length. However, it is advisable to use straight lengths of conduit without obstacles whenever transit times are to be measured.

7 Detection of radioactive tracer

7.1 General

For online radioactive tracer applications, detection system normally consists of a set of gamma radiation detectors connected to a data acquisition system.

7.2 Gamma radiation detector

Gamma detectors have to be effective and ruggedized for rough industrial environments. Solid inorganic scintillators meet most of the requirements. Two types of scintillation detectors, NaI(Tl) and $\text{Bi}_3\text{Ge}_4\text{O}_{12}$ (BGO), are common.

NaI(Tl)-detectors of 2 inch × 2 inch (length × diameter) are mostly used in field measurements. Measurements can be carried out using single channel analysers where all gamma energies between a lower and an upper limit are recorded.

If space is limited, BGO crystals of smaller size 1 inch × 1 inch but with approximately half detection efficiency of 2 inch × 2 inch NaI(Tl)-detectors may be used. Their smaller sizes also need less collimator weight, so they are easier to mount and handle, but somewhat more expensive.

7.3 Detector arrangement

Generally, detection of gamma activity in a detection cross-section can be carried out with only one detector, supposing a sufficiently homogeneous tracer concentration in the detection cross-section of the conduit as shown in [Figure 7](#) (left).

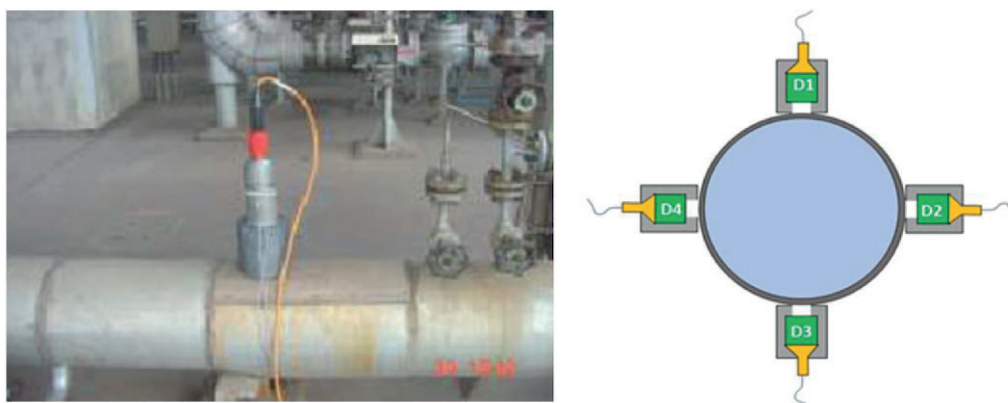


Figure 7 — Typical arrangements of gamma detectors

In order to check the homogeneity of the tracer distribution in the flow, more detectors may be added, for instance in a geometry like that suggested in [Figure 7](#) (right). For a fully homogeneous tracer distribution, all the detectors should give the same reading when corrected for their eventual difference in total counting efficiency.

7.4 Data acquisition system

Radiation detectors are connected to a data acquisition system (DAQ). One, two or more radiation detectors are used for flow rate measurement in closed conduits. The data acquisition system (see [Figure 8](#)), which collects signals from the radiation detectors, is the basic equipment for online radioactive tracer measurements. It ensures collection, treatment and visualization of the data in real time.



Figure 8 — Example of a data acquisition system

8 Procedures for applying radioactive tracer methods

8.1 Transit time method

8.1.1 Location of injection cross-section

The location of injection cross-section depends mainly on the available length of conduit between the injection cross-section and the first detection cross-section. The length of conduit between the injection

cross-section and the first detection cross-section, shall be equal or longer than the theoretical mixing length.

When the available length of conduit between the injection cross-section and the first detection cross-section is less than the theoretical mixing length, it is recommended to inject tracer upstream of a fan, a pump or other turbulence-generating devices.

8.1.2 Pulse injection of radioactive tracer

Pulse injection of radioactive tracer is applied in the transit time method. In order to minimize dispersion of the measured intensity/time distributions, the tracer shall be injected as rapidly as possible, with no “tailing” of the injected tracer from the injection tubes within the conduit. This can be achieved by any of the following means:

- by ensuring that the injected tracer is flushed into the conduit by a flow of radioactive tracer-free material;
- by breaking with a suitable device an ampoule containing the radioactive tracer to be injected in the conduit.

Injection systems are generally home-made, built and adapted for specific applications and field conditions. They vary considerably in design from the simplest (a syringe) to the most complex (devices for remote injection into pressure vessels). [Figure 9](#) presents an example of a radioactive gas tracer injection system for closed conduits and [Figure 10](#) presents an example of a radioactive liquid tracer injection system for closed conduits.

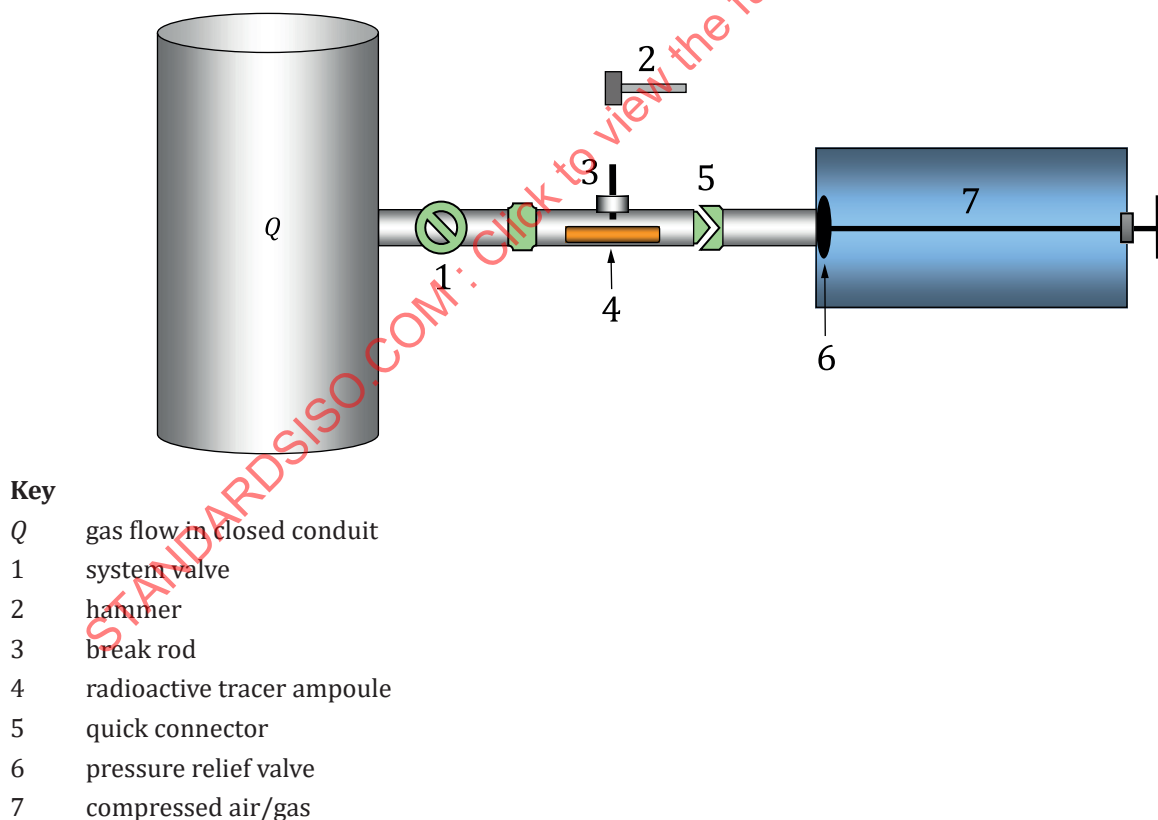
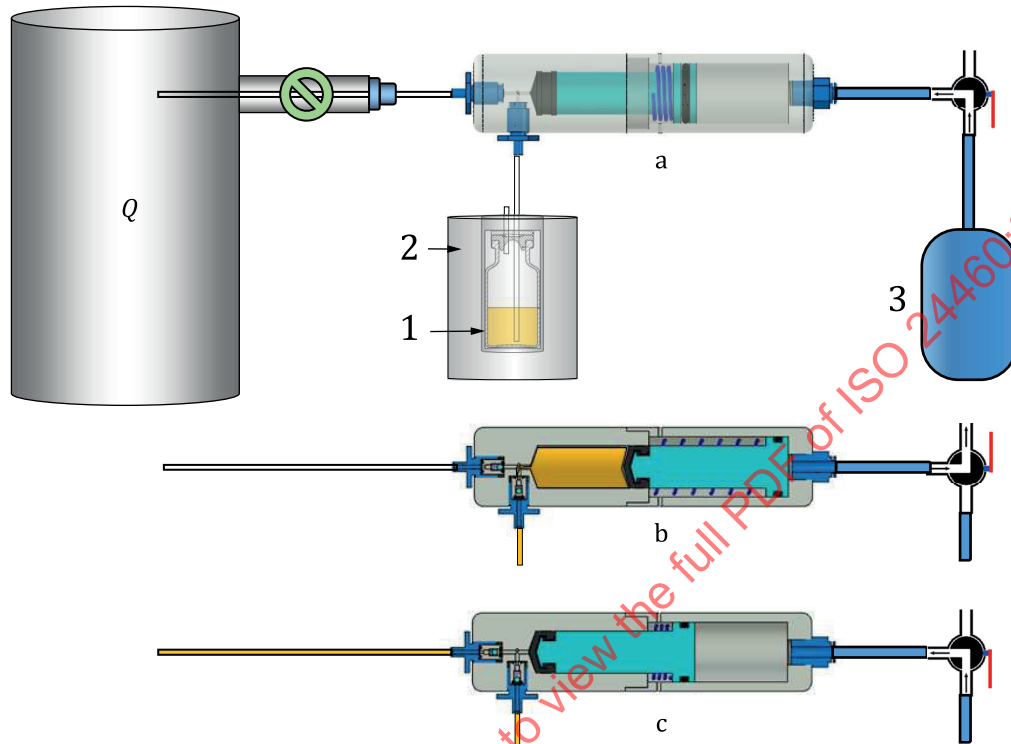


Figure 9 — Example of radioactive gas tracer injection system

The operation procedure is carried out as follows:

- a) insert the ampoule with radioactive gas tracer;
- b) connect the quick connector;

- c) open the system valve;
- d) break the ampoule;
- e) open the pressure release valve;
- f) close the system valve.



Key

- 1 radioactive tracer
- 2 lead container
- 3 compressed air tank
- Q liquid flow in closed conduit
- a Liquid tracer injection setup.
- b Charging of tracer.
- c Injection of tracer.

Figure 10 — Example of radioactive liquid tracer injection system

8.1.3 Estimation of the activity to be injected

The amount of radioactivity is estimated based on efficiency of the detection system, required accuracy, dilution between injection and detection points, and background radiation level.

A simplified calculation method is used to roughly estimating the radioactivity, A , to be injected. For a conservative estimation of radioactivity, it is assumed that the volume, V , of the zone between injection and second detector cross-sections is a perfect mixer with the radioactivity concentration, $C = A/V$. It is recommended to inject an amount of radiotracer to get a radioactivity concentration equal to ten

times the lower detection limit or the minimal detectable concentration of the tracer, C_{\min} , i.e. the radioactivity to be injected is as given in [Formula \(8\)](#):

$$A = 10 C_{\min} V \quad (8)$$

C_{\min} depends on radiation background level, R_B , detection efficiency, ε , and count time, Δt . For a 95 % confidence limit, C_{\min} can be delivered as given in [Formula \(9\)](#):

$$C_{\min} = \frac{2}{\varepsilon} \left(\frac{2R_B}{\Delta t} \right)^{1/2} \quad (9)$$

where

R_B is the background count rate;

Δt is the count time;

ε is the detection efficiency.

Thus, the activity to be injected can be calculated using [Formula \(10\)](#):

$$A = \frac{28}{\varepsilon} \left(\frac{R_B}{\Delta t} \right)^{1/2} \cdot V \quad (10)$$

The detection efficiency is defined as the response (s^{-1}) of the detector to the unit specific activity ($Bq \cdot m^{-3}$) of the fluid inside the closed conduit at a given detection geometry. Its unit is [$s^{-1} \cdot Bq^{-1} \cdot m^3$].

The detection efficiency can be measured experimentally by simulating the field experimental arrangement in the laboratory using a piece of pipe of the same diameter and wall thickness. The pipe is plugged at both ends and an injection port is installed on the pipe. The background count rate is measured at the beginning. The pipe is filled with radiotracer solution with known specific activity and the count rate is measured.

8.1.4 Choice of measuring length

8.1.4.1 Mixing of radioactive tracer

The radioactive tracer shall be sufficiently mixed with the flow before the first detection cross-section for the recorded response curves at both detectors to be adequately representative of the mean flow. The tracer should be injected as rapidly as possible to minimize the longitudinal dispersion of the tracer.

8.1.4.2 Length of conduit between injection and first detector

The length of conduit between the injection and first detector shall be equal to or greater than the mixing length and should preferably contain no pipe fittings or sections likely to significantly increase the longitudinal dispersion of the tracer at the detection cross-sections.

8.1.4.3 Length of conduit between detection cross-sections

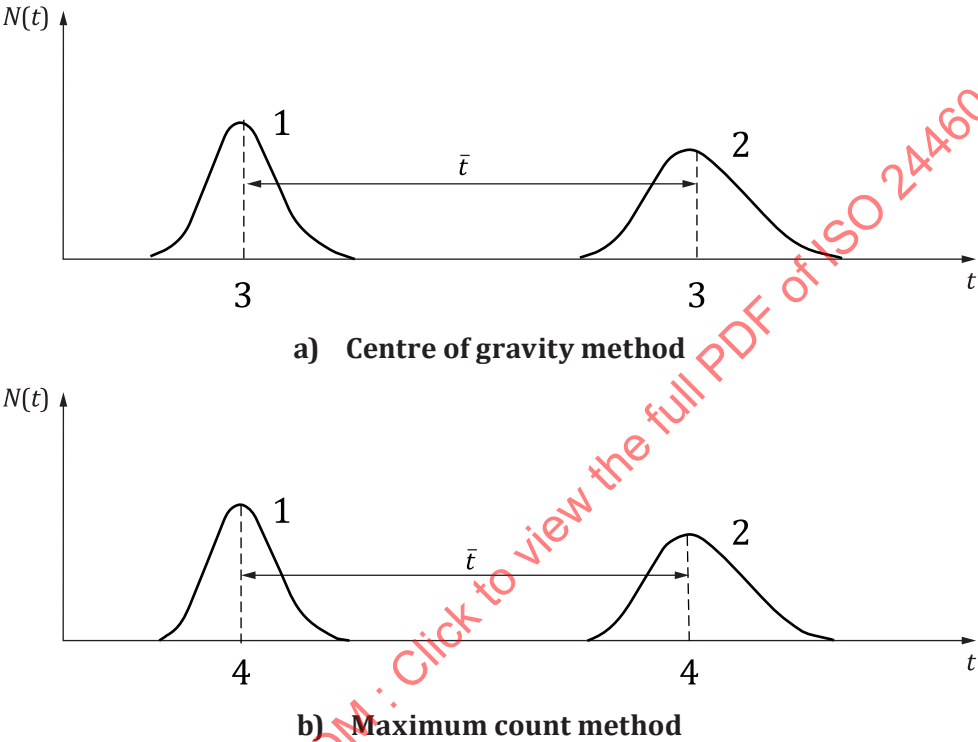
The length of conduit necessary between the detection cross-sections depends on the axial velocity of the fluid, the spatial dispersion of the tracer at the detection cross-sections and the required accuracy of the measurement of transit time.

8.1.5 Calculation of transit time

The transit time is calculated from the two corrected tracer response curves. This procedure is introduced in [Annex A](#).

When preliminary rough calculations are needed, the transit time may be determined from the difference in times corresponding to the following positions on the recorded distribution from the detectors.

- The centre of gravity, i.e. the first moment of the tracer response curve, is the theoretically correct characteristic point in all cases. [Figure 11](#) (a) illustrates \bar{t} between centres of gravity (represented by the first moments) of the two recorded tracer response curves.
- The maximum concentration method is used only when a rapid approximate determination is required. The dotted lines in [Figure 11](#) (b) indicate the times for the maximum of the response curves for the first and second detector.



Key	
1	response curve of first detector
2	response curve of second detector
3	centre of gravity
4	time of the maximum count rate
$N(t)$	count rate
t	time
\bar{t}	transit time

Figure 11 — Characteristic points for calculation of transit time \bar{t}

8.2 Constant rate injection method

8.2.1 Preparation of the radioactive tracer to be injected

It is essential that the radioactive tracer is homogeneously distributed in the fluid to be injected. It is important that the total injected tracer volume has the same tracer concentration (i.e. constant activity concentration, for instance in Bq/ml) throughout the injection period with as constant volumetric injection rate as possible.

To minimize possible adsorption effects with the tracer, it is advisable to add to this solution a carrier constituted by a certain amount of a non-radioactive substance chemically identical to that containing the radioactive tracer.

For some possible tracer compounds, i.e. for instance $^{99m}\text{TcO}_4^-$ (see [Table 2](#)), such a non-radioactive compound does not exist because Tc has no stable isotope. In such cases, a homologue compound may be applied. In the case of $^{99m}\text{TcO}_4^-$, the homologue compound MnO_4^- in the form of KMnO_4 may be applied as a carrier.

8.2.2 Injection of the radioactive tracer

The radioactive tracer shall be injected into the conduit at a constant slow rate and for a sufficient duration to ensure a satisfactory period of constant concentration at the sampling cross-section.

The injection may be made by means of volumetric pumps with a precision of less than 0,5 % to 1 %.

It shall be possible to check the following during implementation of the tracer injection:

- that the injection system is always free from leaks;
- that the injection rate is constant over the whole injection duration.

8.2.3 Measurement of injection rate

Although modern pumping equipment has a high precision for preselected values of pumping rate, the injection system shall always be calibrated before and after the flow experiment. This can be done by pumping a non-radioactive solution of the same nature and properties as the injection solution while collecting consecutive samples with a sample changer. The amount of fluid in each sample is determined volumetrically or by mass. The mean value of pumping rate recorded for the different samples shall be used in the calculation of injection flow rate provided that the two calibrations (before and after the injection) do not differ by more than a value consistent with the overall required accuracy of flow measurement (for example 1 %).

8.3 Integration method

In the integration (or total-count) method, an amount activity of radioactive tracer is injected as a pulse, then this amount shall eventually pass any downstream detection point after mixing length. The total amount of counts registered by the detection system is related with the flow rate. The experimental setup and procedure of the method are introduced in [9.4](#).

9 Uncertainty

9.1 General

9.1.1 Evaluation of uncertainty

As with any measurement of physical quantities, the determination of a flow rate in a closed conduit by radioactive tracer methods is subject to various uncertainties. The evaluation and reporting of the uncertainty of a measured flow rate are essential, especially when the result is used as part of a basis for making decisions or for fiscal metering. Methods for evaluating the standard uncertainties are classified as either Type A or Type B.

- Type A evaluation: Uncertainty estimates obtained as standard deviations by the statistical treatment of repeated measurement results.
- Type B evaluation: Uncertainty estimates obtained from any information other than statistical treatment of repeated measurement results. This can be information from past experience with

measurements, calibration certificates, manufacturer's specifications, calculations, published information, and common sense.

The standard deviation (or uncertainty) of a single radiation count can be estimated by the square root of the count. The Poisson model of radiation counting is the mathematical basis for this rule. The use of this approximation is a Type B evaluation of uncertainty.

9.1.2 Procedures for evaluating the uncertainty of a measured flow rate

The usual steps for evaluating and reporting the uncertainty of a measured flow rate may be summarized as follows (adapted from ISO/IEC Guide 98-3:2008, Clause 8):

- express the mathematical relationship between the flow rate Q and the input quantities X_i ;
- determine x_i , the estimated value of input quantity X_i ;
- evaluate the standard uncertainty u_{x_i} for each input estimate x_i using either a Type A or Type B evaluation;
- evaluate the covariance associated with any input estimates that are correlated;
- calculate the flow rate from the mathematical relationship using the estimates x_i obtained in step b);
- determine the combined standard uncertainty u_Q of the flow rate Q using the uncertainty propagation formulae;
- calculate the extended uncertainty of the flow rate in terms of a coverage factor or a level of confidence;
- report the flow rate and its extended uncertainty with the level of confidence.

9.1.3 Uncertainty propagation formula

If the input estimates x, y, \dots are independent (uncorrelated) and z is the function of the input quantities, i.e. $z = f(x, y, \dots)$, the general uncertainty propagation is given as [Formula \(11\)](#):

$$u_z = \sqrt{\left(\frac{\partial f}{\partial x} u_x\right)^2 + \left(\frac{\partial f}{\partial y} u_y\right)^2 + \dots} \quad (11)$$

where u_x, u_y, \dots and u_z are the standard uncertainties of x, y, \dots and z , respectively.

Some commonly used uncertainty propagation formulae derived from the general formula are shown in [Table 3](#).

Table 3 — Uncertainty propagation formulae

Processes	Functions	Uncertainty propagation formulae
Addition, subtraction	$z = x \pm y \pm \dots$	$u_z = \sqrt{u_x^2 + u_y^2 + \dots}$
Multiplication, division	$z = x \cdot y, z = \frac{x}{y}$	$u_z = z \sqrt{\left(\frac{u_x}{x}\right)^2 + \left(\frac{u_y}{y}\right)^2}$
Multiplication by an exact value	$z = kx + c$	$u_z = k u_x$
Power of a single variable	$z = x^n$	$u_z = n x^{n-1} u_x$
Reciprocal of a single variable	$z = \frac{1}{x}$	$u_z = \frac{u_x}{x^2}$

9.2 Uncertainty of flow rate measured using the transit time method

In the transit time method, the volumetric flow rate Q of fluid flow in closed conduits is calculated using [Formula \(1\)](#). The principle of the flow rate measurement using the transit time method is illustrated in [Figure 1](#).

The formula for the standard uncertainty u_Q of the flow rate can be derived using the uncertainty propagation formula for division:

$$u_Q = Q \cdot \sqrt{\left(\frac{u_V}{V}\right)^2 + \left(\frac{u_{\bar{t}}}{\bar{t}}\right)^2} \quad (12)$$

where

u_V is the standard uncertainty of V ;

$u_{\bar{t}}$ is the standard uncertainty of \bar{t} .

Both the volume V and transit time \bar{t} cannot be measured directly. They are determined from other quantities through certain mathematical relationships. In case the cross-section of the conduit is circular, the volume V is a function of the inner diameter d of the conduit and the measuring length l between the two detection cross-sections. The mathematical relationship is given by [Formula \(13\)](#).

$$V = \frac{\pi}{4} d^2 l \quad (13)$$

[Formula \(14\)](#) for the standard uncertainty of the volume can be derived using the uncertainty propagation formulae in [Table 3](#):

$$u_V = V \cdot \sqrt{\left(\frac{2u_d}{d}\right)^2 + \left(\frac{u_l}{l}\right)^2} \quad (14)$$

where

u_d is the standard uncertainty of d ;

u_l is the standard uncertainty of l .

The standard uncertainties, u_d and u_l , are determined by Type A or Type B evaluations of the values measured or obtained from various sources.

The transit time \bar{t} is the difference of the mean residence times (MRTs) of the two response curves obtained from the two radiation detectors. The process for the calculation of MRTs and their standard uncertainties is quite complex because many data need to be handled and several formulae are involved in the calculation. The process for the calculation of \bar{t} and $u_{\bar{t}}$ is demonstrated in [Annex A](#).

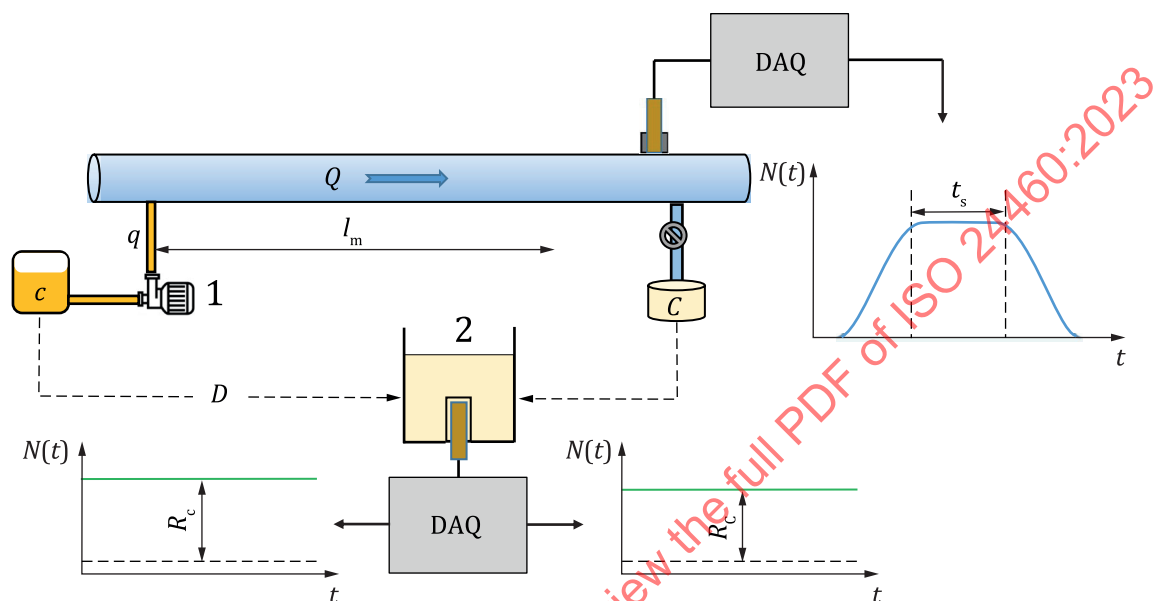
In addition to the uncertainty sources mentioned above, some other possible uncertainty sources should be checked. Some examples are:

- imperfect mixing of tracer with the main flow;
- modified internal shape of the conduit, such as deposition/scaling, erosion/corrosion, air, bubbles;
- unknown flow path, such as leaks, side openings, dead-end piping;
- adsorption of tracer to the conduit wall;
- fluctuation of flow rate.

Every care should be given in conducting experiments to avoid these error sources. If some of them are still existing, reasonable evaluations of their standard uncertainty should be performed and the results should be included in the calculation of the standard uncertainty of the measured flow rate.

9.3 Uncertainty of flow rate measured using the constant rate injection method

In the constant rate injection method, the volumetric flow rate of fluid flow in closed conduits is calculated using [Formula \(6\)](#). The principle of the flow rate measurement using the constant rate injection method is illustrated in [Figure 12](#).



Key

- 1 constant rate injection pump
- 2 Marinelli beaker
- c activity concentration of tracer solution injected
- C activity concentration of the sample collected
- q volumetric flow rate of tracer
- Q volumetric flow rate in closed conduit
- $N(t)$ count rate
- t time
- t_s time for sampling
- R_c net radiation count rate of the diluted injection solution
- R_c net radiation count rate of the collected sample solution
- DAQ data acquisition system
- l_m mixing length
- D dilution factor

Figure 12 — Flow rate measurement using the constant rate injection method

The net radiation count rates are proportional to the radioactive tracer concentrations if their counting efficiencies are identical. To meet this requirement, an aliquot of the injection solution needs to be

diluted to make a measuring solution of similar concentration to the collected solution. Then the ratio of concentration is:

$$Q = q \frac{c}{C} = q \frac{DR_c}{R_c} = q \frac{D \left(\frac{N_c}{t_c} - \frac{N_B}{t_B} \right)}{\left(\frac{N_C}{t_C} - \frac{N_B}{t_B} \right)} \quad (15)$$

where

- q is the volumetric flow rate of tracer;
- Q is the volumetric flow rate in closed conduit;
- c is the activity concentration of tracer solution injected;
- C is the activity concentration of the sample collected;
- R_c is the net radiation count rate of the diluted injection solution;
- R_C is the net radiation count rate of the collected solution;
- D is the dilution factor of the injection solution;
- N_c is the radiation count measured with the diluted solution during a count time t_c ;
- N_C is the radiation count measured with the collected solution during a count time t_C ;
- N_B is the radiation count measured with tracer free solution during a count time t_B .

For simplicity of calculation, use a same count time for all radiation measurements, i.e.

$$t_c = t_C = t_B$$

Then the above flow rate formulae can be modified as follows:

$$Q = \frac{qD(N_c - N_B)}{N_C - N_B} \quad (16)$$

The formula for the standard uncertainty of the volume flow rate can be derived using the uncertainty propagation formulae in [Table 3](#).

$$u_Q = Q \sqrt{\left(\frac{u_q}{q} \right)^2 + \left(\frac{u_D}{D} \right)^2 + \left(\frac{u_{N_c - N_B}}{N_c - N_B} \right)^2 + \left(\frac{u_{N_C - N_B}}{N_C - N_B} \right)^2} \quad (17)$$

where

- u_q is the standard uncertainty of q ;
- u_D is the standard uncertainty of D ;
- $u_{N_c - N_B}$ is the standard uncertainty of $N_c - N_B$;
- $u_{N_C - N_B}$ is the standard uncertainty of $N_C - N_B$.

$$\text{As } u_{N_c} = \sqrt{N_c} \text{ and } u_{N_B} = \sqrt{N_B}, u_{N_c - N_B} = \sqrt{(u_{N_c})^2 + (u_{N_B})^2} = \sqrt{N_c + N_B}$$

$$\text{As } u_{N_C} = \sqrt{N_C} \text{ and } u_{N_B} = \sqrt{N_B}, u_{N_C - N_B} = \sqrt{(u_{N_C})^2 + (u_{N_B})^2} = \sqrt{N_C + N_B}$$

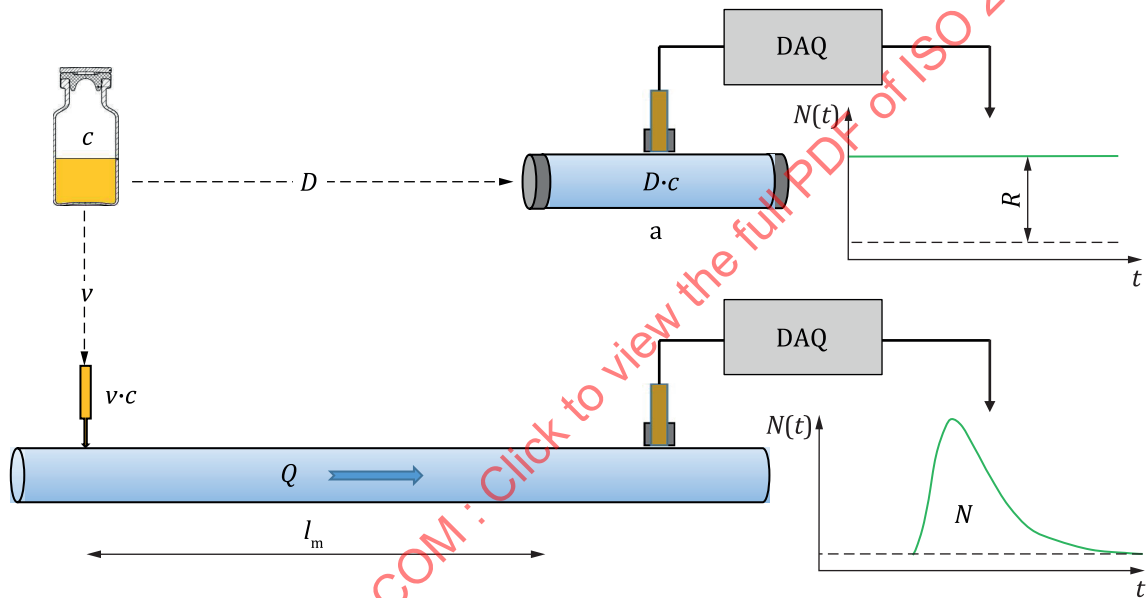
Therefore,

$$u_Q = Q \cdot \sqrt{\left(\frac{u_q}{Q}\right)^2 + \left(\frac{u_D}{D}\right)^2 + \frac{N_c + N_B}{(N_c - N_B)^2} + \frac{N_c + N_B}{(N_c - N_B)^2}} \quad (18)$$

The standard uncertainty of the volume flow rate of injected tracer u_q can be determined by Type A and/or Type B evaluations of the values measured or obtained from various sources. The standard uncertainty of the dilution factor u_D can be determined based on the uncertainties of volume measurements in the dilution process. The radiation counts, N_c , N_c and N_B , are measured at a same condition and with a same count time.

9.4 Uncertainty of flow rate measured using the integration method

In the integration method, the volumetric flow rate of fluid flow in closed conduits is calculated using [Formula \(7\)](#). The principle of the flow rate measurement using the integration method is illustrated in [Figure 13](#).



Key

- c activity concentration of radioactive tracer
- Q volumetric flow rate in closed conduit
- t time
- $N(t)$ measured radiation count rate
- R net radiation count rate of the diluted injection solution
- DAQ data acquisition system
- l_m mixing length
- v volume of injected tracer
- $D \cdot c$ activity concentration of the diluted injection solution
- $v \cdot c$ total activity of injected tracer
- D dilution factor
- N integrated net radiation count
- a Setup for calibration.

Figure 13 — Flow rate measurement using the integration method

An experimental procedure to obtain calibration factor, F , is as follows.

After setting the calibration equipment, record the background count N_b for a count time t_b . Prepare a calibration solution by diluting (dilution factor is D) an aliquot of the tracer solution (activity concentration is c) to be injected. Fill the pipe with the calibration solution and record the radiation count N_g for a count time t_g .

For the simplicity of calculations, it is assumed that the effects of dead time and decay are negligible. Then, the net count rate R and the concentration of the calibration solution C are:

$$R = \frac{N_g}{t_g} - \frac{N_b}{t_b} \quad (19)$$

$$C = Dc \quad (20)$$

The calibration factor F is the proportional coefficient between R and C .

$$R = FC \quad (21)$$

If the volume of the tracer solution injected for a flow rate measurement is V , then the total injected activity A is:

$$A = vc \quad (22)$$

Then,

$$FA = \frac{R}{C} \cdot vc = \frac{R}{Dc} \cdot vc = \frac{Rv}{D} \quad (23)$$

Therefore, the calculation for volumetric flow rate using the measured values is given by [Formula \(24\)](#):

$$Q = \frac{Rv}{DN} \quad (24)$$

[Formula \(25\)](#) for the standard uncertainty of the volumetric flow rate can be derived using the uncertainty propagation formulae for multiplication and division:

$$u_Q = Q \cdot \sqrt{\left(\frac{u_R}{R}\right)^2 + \left(\frac{u_v}{v}\right)^2 + \left(\frac{u_D}{D}\right)^2 + \left(\frac{u_N}{N}\right)^2} \quad (25)$$

where

u_R is the standard uncertainty of R ;

u_v is the standard uncertainty of v ;

u_D is the standard uncertainty of D ;

u_N is the standard uncertainty of N .

As $u_N = \sqrt{N}$,

$$u_Q = Q \cdot \sqrt{\left(\frac{u_R}{R}\right)^2 + \left(\frac{u_v}{v}\right)^2 + \left(\frac{u_D}{D}\right)^2 + \frac{1}{N}} \quad (26)$$

If a radiation count N_i is measured during a count time t_i , then the count rate is N_i/t_i and its standard uncertainty is $\sqrt{N_i}/t_i$. Using the uncertainty propagation formula for subtraction, the standard uncertainty of the net count rate R is derived as follows:

$$u_R = \sqrt{\frac{N_g}{(t_g)^2} + \frac{N_b}{(t_b)^2}} \quad (27)$$

The standard uncertainty of the volume of injected tracer solution u_v and the standard uncertainty of the dilution factor u_D can be determined based on the uncertainties of volume measurements in the injection and dilution processes. N is the dead time, background, and decay corrected value of the total radiation count integrated during the passage of the radioactive tracer cloud through the detection cross-section.

Annex A (informative)

Calculation of transit time and its standard uncertainty

A.1 General

In the transit time method, a tracer is injected instantaneously into the upstream in a closed conduit, and radiation counts are measured at two detection points to obtain response curves, which are radiation count (N_i) vs. time (t_i) curves.

The transit time is the difference between the two mean residence times (MRTs), which are calculated from the two response curves. Before the MRT calculations, the measured radiation counts should be calibrated to get more accurate results. It includes dead time, background, decay, and tail correction.

As radioactive decay is a random process, a radiation count N_i is described by a Poisson distribution, and the quantity of the standard uncertainty of the count is approximately the square root of the count, as given by [Formula \(A.1\)](#):

$$u_{N_i} = \sqrt{N_i} \quad (\text{A.1})$$

The standard uncertainties of the corrected count, MRT, and transit time are calculated using the uncertainty propagation formulae in [Table 3](#).

A.2 Corrections of radiation counts and propagation of uncertainties

A.2.1 Dead time correction

The dead time corrected count (N'_i) is given as [Formula \(A.2\)](#):

$$N'_i = \frac{N_i}{1 - t_d \frac{N_i}{\Delta t}} \quad (\text{A.2})$$

where

- N_i is the measured gross count;
- t_d is the dead time of the detector;
- Δt is the count time.

To facilitate the derivation of the formula for standard uncertainty ($u_{N_i'}$), [Formula \(A.2\)](#) is rearranged as [Formula \(A.3\)](#):

$$N_i' = \frac{1}{\frac{1}{N_i} - \frac{t_d}{\Delta t}} \quad (\text{A.3})$$

Using the uncertainty propagation formula for reciprocal of a single variable and [Formula \(A.1\)](#), as given in [Formula \(A.4\)](#):

$$u_{\frac{1}{N_i}} = \frac{u_{N_i}}{N_i^2} = \frac{\sqrt{N_i}}{N_i^2} = \frac{1}{N_i \sqrt{N_i}} \quad (\text{A.4})$$

The uncertainty of the count time is negligible when modern data acquisition system is used. The dead time with very low uncertainty can be obtained by using a long counting time in the two-source method for dead time evaluation. Therefore, $\frac{t_d}{\Delta t}$ can be considered as an exact value. Using the uncertainty propagation formula for subtraction by an exact value as given in [Formula \(A.5\)](#):

$$u_{\frac{1}{N_i} - \frac{t_d}{\Delta t}} = \frac{1}{N_i \sqrt{N_i}} \quad (\text{A.5})$$

Using the uncertainty propagation formula for reciprocal of a single variable, the standard uncertainty of the dead time corrected count can be derived as given in [Formula \(A.6\)](#):

$$u_{N_i'} = u_{\frac{1}{\frac{1}{N_i} - \frac{t_d}{\Delta t}}} = \frac{\frac{1}{N_i \sqrt{N_i}}}{\left(\frac{1}{N_i} - \frac{t_d}{\Delta t}\right)^2} = \frac{\sqrt{N_i}}{\left(1 - \frac{t_d}{\Delta t} N_i\right)^2} \quad (\text{A.6})$$

A.2.2 Background correction

The background corrected count (N_i'') is given by [Formula \(A.7\)](#):

$$N_i'' = N_i' - B_i \quad (\text{A.7})$$

where B_i is the background count measured during the count time Δt .

Using the uncertainty propagation formula for subtraction, the standard uncertainty of the background corrected count can be derived as given in [Formula \(A.8\)](#):

$$u_{N_i''} = \sqrt{\left(u_{N_i'}\right)^2 + \left(u_{B_i}\right)^2} \quad (\text{A.8})$$

where u_{B_i} is the uncertainty of the background count, $u_{B_i} = \sqrt{B_i}$, therefore the standard uncertainty of the background corrected count can be derived as given in [Formula \(A.9\)](#):

$$u_{N_i''} = \sqrt{\left(u_{N_i'}\right)^2 + B_i} \quad (\text{A.9})$$

In most cases, the level of background counts is significant due to the natural background radiation and scattered radiation from the radioactive tracer used. Therefore, background correction is indispensable for the accurate calculation of MRT and its uncertainty.